Introduction

The purpose of this paper is to report on a study that used concept maps for measuring change in teachers' mathematical content knowledge in algebra. The study was conducted as part of a larger research program that examines the effects of a 5-year National Science Foundation teacher retention and renewal project in mathematics on its participating teachers and schools. The project develops the leadership capacity of experienced teachers in nine Southern Californian school districts to support them in working with beginning teachers (those in their first five years of teaching) in their schools. Thus far the project has impacted 380 beginning and experienced teachers.

The project worked with two cohorts of teacher leaders. The first cohort started with summer institutes in 2001 and the second in 2003. During these two-week summer institutes, participants engaged in investigative project-based mathematics activities designed to enhance their understandings of algebra, develop their mathematics and pedagogical knowledge, increase their capacity to teach mathematics to diverse student populations, and address inequities in education. This study was conducted with the second cohort of teacher leaders to examine how their understandings of algebra changed as a result of participation in a two-week Summer Institute. In particular we asked:

1. What understandings did teachers have at the beginning of the institute?
2. What were teachers' understandings at the end of the institute?
3. How did these understandings change?

Previous Uses of Concept Maps

A concept map (Novak, 1984), is a two-dimensional image that is used to represent the relationships among a learner's concepts related to a central theme or topic. Individual ideas or concepts on the map are represented by labeled circles or boxes that are connected by lines that represent a link or relationship in the respondent's mind between the concepts. The respondent builds his or her map around a central theme or topic and new topics are added in a hierarchical network. Concepts most closely related to the main topic are closely linked to the main concept and concepts not closely related are linked further away. Concept maps have been used by mathematics educators as a way of teaching subject matter to students and prospective teachers (Bolte, 1999), as a means of identifying misconceptions (Huerta, Galan, & Granell, 2003) and as an assessment instrument (Bolte, 1999). Science educators have used them extensively as a tool for formative student assessment, for identifying student knowledge prior to instruction and for looking at change in knowledge as a result of instruction (Jones, Carter, & Rua, 2000, 1998; Novak, 1998; Novak, 1991). Novak (1984) wrote that they were "developed specifically to tap into a science learner's cognitive structure and to externalize, for both the learner and the teacher to see, what the learner already knows" (p. 40). The project leaders decided to use concept maps to assess change in teachers' understandings of mathematics subject matter because they are in line with the overall philosophy of the project and they capture the types of mathematical learning that the project values. In this paper we describe the project's perspective on mathematics learning and assessment, the use of concept maps, and the results.

The Project's Perspective on Mathematics Learning

The project upholds the view that mathematics *is* an exciting and beautiful discipline that is important for all people to experience. Further, it is important for all people to have the power of mathematics at their disposal, both for their enjoyment and for use as a problem-solving tool.

Doing Mathematics

The active "doing" of mathematics is an important component of this project's professional development for teachers, often whom, being the recipients of a more traditional mathematics education focused on following procedures and memorizing formulas, are unfamiliar with mathematical investigations.

> *…anyone who has done mathematics knows what comes first--a problem. Sometimes a problem is called a conjecture…In developing and understanding a subject, axioms come late. Then in formal presentation they come early…Examples, problems and solutions come first…The view that mathematics is in essence derived from axioms is backward. In fact, it's wrong*. (Hersh, 1987, p. 6)

The view of the discipline of mathematics as one that is derived from axioms is prevalent in traditional K-12 classrooms. In such classrooms, students learn rules and procedures presented by the teacher, are expected to memorize these rules, and spend many hours practicing using them. Students in these classrooms often come away with the impression that doing mathematics is a rote and mechanical activity, all authority for mathematics lies in a textbook or as privileged knowledge inherent in the classroom teacher, and that memorization of formulas and procedures is the way to succeed mathematically (Hough, 2001; Thompson, Philipp, Thompson, Boyd, 1994). As a result, many students turn away from mathematics early; they often see it as a discipline understandable by a select few and choose not to pursue higher mathematics (Skemp, 1987).

In contrast, this project promotes mathematics as a discipline that is useful and beautiful—and the doing of mathematics as a creative, sense making activity that entails interpretation, hypothesizing, effort, and exploration. This view of mathematics is in line with the type of investigative mathematics envisioned in the *Principles and Standards for School Mathematics* (NCTM, 2000). NCTM emphasizes the considerable importance of the classroom teacher in implementing this type of mathematics learning: "Teachers are key figures in changing the ways in which mathematics is taught and learned in school" (p. 2). Goldsmith and Schifter recognize, like many mathematics researchers, that such change is fraught with difficulty.

> *Because this picture [of mathematics] is a significant departure from traditional instruction, teachers cannot realize these new images by simply adjusting a bit of practice here and there or by importing a new teaching technique or curriculum package. For many teachers these changes involve reconstituting fundamental notions of teaching, learning and mathematics.* (Goldsmith & Schifter, 1997 p.20)

The project recognizes that teachers, many of whom were recipients of a traditional mathematics programs, need many of these opportunities to re-learn the mathematics that they are to teach.

Learning Important Mathematics Content

In addition to reconstructing notions of “doing” mathematics, there is the issue of learning and re-learning mathematics content and making connections among subject matter topics. According to Ball (2000), true mathematical content understanding includes knowledge about mathematics, how mathematical solutions are justified, and how conjectures are proved or disproved, in addition to a cohesive and interconnected understanding of subject matter—both the procedures and concepts of the subject and the relationships among them. Ma (1999) writes extensively about interconnectivity of concepts in mathematics in her descriptions of “profound understanding of fundamental mathematics,” which is a deep, vast and thorough understanding of mathematics subject matter that is achieved when it is connected in the learner’s mind to other conceptual ideas related to the topic. We think of these connections between related concepts in the learner’s mind as knowledge networks. How does one learn such an understanding of mathematics that is much more than the memorization of facts and procedures? We define learning as the process of taking in new information from the environment, comparing and contrasting it to past experience and previously understood information, and evaluating, organizing and storing the acquired information so that it is available for use in new situations (Weissglass, 1994). In the project, teachers are offered extensive opportunities to participate in mathematics activities that allow them to extend their current understanding of particular subject matter and apply that knowledge to authentic problem situations, which enhance the development of deeper and more connected mathematics knowledge networks. These networks or *cognitive schemas* that exist in the learner's mind are the principle determining factors for how new ideas and concepts will be constructed. To capture participants' changes in their knowledge networks about algebra, we asked teachers to draw concept maps at the beginning and the end of the Summer Institute.

Assessment of Teachers' Mathematical Understandings

The social and psychological dimensions of educational change, are areas that are often neglected in professional support and development projects (Weissglass, 1994). Rarely does professional development address teacher's beliefs values and emotions. The project leadership recognizes and addresses the importance of these dimensions and thus strives to develop a safe, trusting, professional learning community. When a study to assess change in these teacher leaders' mathematical understandings was devised, the project leaders wanted an assessment tool that was consistent with the five principles (see Figure 1 below) that reflect the underlying philosophy of the project.

Insert Figure 1 here

Project leaders have learned from their years of experience working with teachers of grades K-12 in mathematics that many educators lack confidence in their understanding of mathematics. Typically, teachers' experiences learning mathematics were procedural with little attention to developing conceptual understanding. Even those who were successful in mastering the procedures often feel inadequate with regard to the underlying mathematical ideas. This lack of confidence creates anxiety and can lead to a situation where teachers resist sharing their ideas for fear of showing a lack of understanding. Assessment of teacher knowledge can magnify this anxiety, which then undermines the development of a safe and respectful learning community. The project leaders chose to use concept maps to assess the learning that took place during the two-week summer institute as they believe their use to be consistent with the five *Principles of Effective and Equitable Assessments*.

Concept maps, used as a pre-assessment and discussed in a non-threatening manner, enabled teachers to reflect on their understandings of algebra and to share their understandings with the researchers and their colleagues in ways that did not interfere with building a safe and trusting learning community (Principle 1). In constructing their concept map, teachers began by reflecting on their existing knowledge of algebra and made decisions about how to represent their ideas in a map. This communicated respect for their thinking and validated the knowledge they brought to the institute (Principle 3). Just as good assessment should enhance students' learning and be an integral part of mathematics instruction (NCTM, 2000), the process of constructing, reflecting on, and discussing pre and post maps - and then reflecting on what was learned over time – was a valuable learning experience for teachers (Principle 2). In addition, concept maps challenged teachers to make connections between the mathematical ideas or concepts that they hold, thus communicating that mathematical knowledge is more than discrete facts and skills (Principle 4). Using concept maps as learners provided teachers with experience and other models for ways in which they might learn about their students' mathematical understanding and promote in students the habit of self-assessment. This contradicts a more narrow view of mathematics learning and the tendency for teachers to use tests as their only means of student assessment, particularly in the current climate of high-stakes tests and accountability (Principle 5).

The project personnel chose not to use multiple-choice or short response tests, as they believe that they do not satisfy these principles. Even task based assessments that require active participation followed by participant discussion about their thinking, which could be a valuable learning experience (Principle 2), can introduce anxiety and competition and thereby interfere with building a trusting community (Principle 1).

Methodology

This study examined how teacher leaders' understandings of algebra changed as a result of participation in a two-week Summer Institute. In particular, we ask the following questions:

1. What understandings did teachers have at the beginning of the institute?
2. What were teachers' understandings at the end of the institute?
3. How did these understandings change?

Context of the Study

The setting for this study was a ten-day summer institute in which experienced teachers came to learn more mathematics content and pedagogy to prepare them to mentor beginning teachers back in their schools and districts. The content focus for the institute was algebra, specifically the concepts of pattern and relationships, families of functions, and their graphs. For part of each day of the institute participants would engage in activities that both built upon their current understandings of and helped build connections between these concepts. For example, in one of the early institute activities participants used pattern blocks to build shapes made out of 1, then 2, then 3, etc. equilateral triangles, finding the perimeter in each case. They used "T-charts" to organize their information, and were asked to identify a pattern to predict what the perimeter would be for 100 triangles and then to generalize this for *x* triangles (verbally and – if they were ready —algebraically). Participants did the same for squares and regular hexagons and then were asked to compare the rules or formulas they devised and were challenged to generalize this pattern for any regular polygon. During this learning activity, participants used a geometric context to identify and describe patterns that they saw in a variety of ways, while being given the opportunity to incorporate formalized algebra if they so desired. Teachers worked collaboratively and shared their ideas and discoveries. Subsequent activities included work with other geometric patterns and describing them algebraically, making graphs of the patterns they identified, discussing why certain relationships were/were not functions, and interpreting graphs. They also had experience with quadratic and exponential functions, and applied what they were learning to real-world data with population growth. Each day's mathematics built upon the prior activities, allowing participants (a group with a wide range of experiences with algebra) to make connections to their prior learning and to develop their mathematical understanding. In addition to this, there were breakout sessions for grade level groups where teachers focused on specific mathematics taught at their grade level and related to what they were learning in the whole-group mathematics sessions.

Participants

Twenty-nine teacher leaders from nine southern California school districts (representing grades K-12) participated in the Summer Institute. Due to absences on two days of data collection we collected twenty-five pre and post response pairs, which were subsequently used in the study.

Data Collection

Pre and post data were collected for this study.

Data collection of pre concept maps. We collected pre-institute concept map data on the first day of the ten-day institute in order to gauge participant's understanding of algebra pre institute. A project leader introduced the concept map activity and explained its purpose: a) to enable participants to reflect on their understanding of algebra, b) to introduce an organizational tool for representing knowledge that is useful as a classroom assessment tool, and c) to use the concept map as a vehicle for assessing the growth in participants' understanding of the mathematical content in order to evaluate the learning opportunities provided during the two-week professional development institute.

The project leader led the participants in creating a concept map about *teenagers,* a topic that all participants enjoyed and which illustrated the different types of organizational patterns (i.e., spider maps, hierarchical maps). The project leader then presented a more complex concept map of fractions. During the introduction, it was stressed that the concept map activity was not an individual assessment, but a program evaluation. To that end, teacher participants were asked to choose a pseudonym to use on the pre- and post-institute concept map; pseudonyms were not matched to teacher participants' identities.

Teachers were given 12 minutes to draw and complete a concept map about *algebra*, the mathematical topic for the two-week institute. The project leader asked participants to use lines to connect words and phrases to the central theme or to previously existing nodes, to connect one idea to another with links, and to write down ideas quickly without judgment, printing legibly.
Time limits were set in order to keep the pre and post data gathering sessions consistent. The particular limit of 12 minutes was based on the previous experience of the project leaders. One of the 25 teacher participants requested and was given more time to complete the concept map. The teacher participants then paired up to discuss the following questions: "What did you learn during this activity?" "What did the activity tell you about how you organize knowledge of algebra?" After pairing up, small groups and the whole group discussed: "What have you learned about algebra?" "How have you organized your ideas about algebra in relation to how others see it?" and "What do you think about using concept maps as an instructional and assessment tool?" The introduction, creation, and discussion of concept maps took approximately 55 minutes.

Data collection of post maps. On the eighth day of the institute, participants drew a second concept map about algebra in the allotted time of 12 minutes in order to capture their understandings of algebra after the institute. At this time they were asked not to refer to the maps that they had created on the first day so that a true comparison of the pre and post maps could occur.

Open-ended Reflections. After participants had created their post maps they discussed and wrote responses to the following prompts:

1. Compare the map you have just finished to your pre institute concept map. What do they show about your growth in learning? What do they not show?

2. a) Choose a concept that appears on both maps. Write a paragraph or two explaining what more you understand about the concept than before you began the institute.

   b) If possible, choose a concept that is on your second map but not on your first. Write a paragraph or two explaining what you now understand about that concept.

The authors analyzed the maps of the twenty-five teacher leaders who completed both a pre and post-institute concept map of algebra. Participant's reflective writings were also analyzed with the intent of triangulating and thus validating the findings derived from the concept maps.

Analysis and Results

There are several ways of analyzing concept maps. In this study we used two analytical methods that are outlined in Novak & Gowind (1984) and Morine-Dershimer (1993): a structural/numerical analysis and a content analysis. We first define the terms used in concept map analysis and use a typical participant map for illustration. A distinct vocabulary has emerged in the literature on concept map analysis, which is summarized in Figure 2.

Insert Figure 2 here

While the number of concepts on a map may be thought of as assessing the breadth of a participant's mathematical understanding of algebraic concepts, the depth and width are used in prior studies to depict the complexity (Winitzky, Kauchack & Kelly, 1995). The width and depth of a map are often added to give a Hierarchical Structure Score (HSS). The higher the HSS score, then, the greater complexity of understanding. The connectedness of a participant's understanding can be represented by the number of chunks and the number of crosslinks on their map (Novak, 1984). Chunks show to what extent a participant groups and orders concepts, whereas a crosslink between two of these subject matter chunks is interpreted as showing a subject's ability to connect thoughts, demonstrating deeper understanding of how topics are interrelated.

We thus define a number of variables that are created by assigning numeric values to these structural components of a concept map. We focus on six such variables that are associated with the definitions in Figure 2 and are depicted in Figure 3.

Insert Figure 3 here

These variables are discussed further using the randomly selected concept map depicted in Figure 4 below for illustration.

Insert Figure 4 here

The map in Figure 4 has 14 concepts, a depth of 3, a width of 6 and HSS 9. On level 1 there are 5 concepts: inverse, number, puzzle, graph, and Mr. Barker. On level 2 there are 6 concepts: both sides, equations, missing number, solve, coordinates and homework. Therefore, the width is 6. On level 3 there are 3 concepts: x, y and high school. This map has only 1 chunk. The cluster of concepts starting at *Puzzle* is a chunk; the two concepts Missing Number and Solve, are connected to *Puzzle*.

To demonstrate how concept maps capture the growth experienced by this participant during the institute we calculated these variables again for this participant's post map that is displayed in Figure 5.

Insert Figure 5 here

The number of concepts on this map has more than doubled to 30 and the depth and width have increased to 4 and 17 respectively, giving a total HSS score of 21. Hence, although the depth of the map is increased by one, the breadth of the map, which has more than doubled, is more than the total number of concepts on the pre-institute map and the difference between HSS pre and post institute is 12. In terms of the overall connectivity of the post map, the number of chunks (5) and the addition of 5 crosslinks between, for example, variables and pattern, show added understandings of relationships among concepts. Table 1 compares the Structural Variables for both the pre- and post-concept maps.

Insert Table 1 here

Missing number, *both sides* and *equation*, concepts that are on the map pre but not on the map post institute, are how a person remembering a procedural approach to algebra might respond. The participant herself said the following:

> *My first map reflects what I remember from high school algebra. One concept that appeared on both maps was solve. I still think algebra involves solving a problem as the ultimate goal but now I understand that getting to the right answer through exploration is important. There still is a right answer but not necessarily one right way the get that answer. My second map reflects the new things that I learned during this week.*

For this participant the comparison of concept maps pre and post institute tells us more than that she has grown in the amount of concepts she can include on her algebra knowledge network and that the complexity of the map is greater post institute. It appears that the mathematics experiences during the institute encouraged her to adapt her knowledge network to accommodate new or re-learned concepts. It is interesting to note that terms not directly related to the subject matter (i.e., Mr. Barker, Homework, High School) do not appear on the participant's post map. This was quite typical for pre concept maps—there were more non-mathematical terms on the pre maps than on the post maps. As such, the authors decided to count the non-mathematical terms as part of a participant's structural variables thus erring on the side of under reporting the extent of growth in these variables from pre to post institute.

Results of Structural Analysis

We next conducted a quantitative analysis to explore the variables for participants both before and after the institute so that a comparison could be made between their two maps. First, we calculated the means, standard deviations, and percent change in mean scores for each participant's pre- and post-concept map. Hence, table 2 below displays these descriptive statistics for participants at the start and at the end of the institute.

Insert Table 2 here

It is clear that the average participant's structural variables increased from pre to post institute. For the variables Concept Number, HSS, Chunk Number and Crosslink Number, the percent change in mean scores were 63.6%, 24.7%, 109.5% and 73.7%, respectively. Second, there is large variance among participants' maps, which is reflected in the wide ranges for each variable. Difference scores and their ranges were calculated to determine if change occurred without regard to the participant's original score for any of the structural variables.

Insert Table 3 here

Table 3 informs us that, although participants started at different places in their understanding of algebra in terms of the amount of algebraic vocabulary they recalled, and the complexity and connectedness of their schemata, on average they grew in their understanding after their participation in the institute. The range of difference scores illustrates that while some participants decreased slightly on these variables, others increased dramatically. Subsequently, a multivariate ANOVA was performed using Concept Number, HSS, Chunk Number and Crosslink Number as dependent variables to test for statistical significance in the pre/post change observed in teachers' understanding of algebra as shown on concept maps. A significant multivariate overall effect was observed ($F(4, 52) = 6.48$; $p< .01$; $\eta^2=.35$) with subsequent F tests showing that all the variables were contributing to this difference. These results are presented in Table 4 and indicate that structurally, large differences between pre and post maps exist in teachers' knowledge networks for algebra.

Insert Table 4 here

Statistically speaking, a univariate partial eta square (η^2) of .01 indicates a small effect size, an eta square of .06 a medium effect size and a partial eta square of .14 indicates a large univariate effect size (Stevens, 1996), hence we claim that the pre/post changes in our participants' concept map variables are indicative of change or growth of algebraic content knowledge. To help verify this claim, we analyzed, using the constant comparative method (Weber, 1990), participants' responses to the prompt: *Compare the map you have just finished to your first concept map. What do they show about your growth in learning? What do they fail to show?* Sixty-nine percent of participants mentioned their obvious increase in the number of concepts between their pre and post maps.

> *"The difference between my two maps is obvious visually. There was a definite increase in vocabulary. There was a significant increase in number of nodes and connections."*

These participants attributed the change to their increased algebra vocabulary.

Increase in complexity. The increase in depth and complexity of maps, represented numerically by the Hierarchical Structure Scores, was verified from the content analysis.

> *My new map illustrates the renewed understanding I have after this institute. The new map is more detailed, complex and positive.*

Almost all of the participants who talked of an increase in concepts also mentioned their post maps being more complex or detailed.

> *The maps show a deeper understanding of the concepts and ideas.*

> *I feel now I have a better understanding of the algebraic concepts [on my first map] and the connections between them.*

Making connections between topics. Forty-one percent of the participants talked of greater connections between algebraic components and connections to other areas of mathematics as being a salient feature of their post map compared to the one they completed at the beginning of the institute.

> *"The maps show how much more connected my second map is and my new understanding of algebra as connected to other areas of math. As a teacher, I think that my own difficulties and fears showed in my first map in my presenting separate units as distinct from one another."*

These connections are represented in our numerical analysis by an increase in the number of chunks and the number of crosslinks.

Results of Content Analysis of Maps

In the previous section we showed that the structure of participant's knowledge networks about algebra changed as a result of participating in the institute. In particular we know that the average participant had more concepts, chunks and links on their post map than on their pre map. In this section we explore how the particular mathematics concepts in that knowledge network changed. Participants' mathematics content experience on entering the institute was mixed, as the project spans K-12 grade levels. For instance, some participants entered the institute with extensive and connected understandings of graphing, formulas and equations (see Figure 6 below). Others, more typically, entered the project with a less extensive understanding (see, for example, Figure 4) and the maps that they subsequently drew showed tremendous growth of content chunks after two weeks of participation in the institute. We chose not to distinguish between the maps of participants based on their grade level or level of mathematical expertise for several important reasons, the first being anonymity. We did not want project leaders to be able to associate names to maps by differentiating by grade level and expertise would mean that this was not the case. Secondly, we believe that it is more appropriate to the aims of the project to look for growth in understanding regardless of where a participant starts out and finishes up.

One way to analyze a participant's subject matter knowledge is to look at the root nodes of the chunks they have formed. For instance in Figure 6 the chunks are: *formulas*, *equations*, *functions*, *graphing, solving*, *word problems, systems, matrices, parabolas, factoring, study skills, and integers*.

Insert Figure 6 here

A content analysis (Weber, 1990) was conducted using the concepts from the maps. Both a priori and emergent category types were used in this analysis. Two super-ordinate a priori categories, *subject matter* and *affective*, were used to sort concepts in each map into the different components of mathematics. For example, the terms *function* and *variable* are clearly mathematical concepts and hence are sorted into the *subject matter* category, whereas the words *boring* and *fun* are attitudes about the subject matter and are sorted in the *affective* category.

Next, using only the concepts in the *subject matter* category, emergent sub-categories according to the chunks shown on the maps were produced. Content chunks were listed for each respondent and were included in Table 5 if they appeared on the list of four or more respondents.

Twelve content chunks were salient on participants' maps (occurred on 5 or more maps). To test for significant differences between the numbers of participants with the salient chunks on their maps pre and post institute a Chi-square analyses was conducted. Column three in table 5 indicates the chi-square values and column 4 the corresponding p-values. Bolded items were statistically significant. Looking at table 5 we see that participants had much more chunks in common at the end of the institute than at the beginning. Table 5 illustrates that the maps became more similar to each other at the end of the institute in addition to Graphing Function, Patterns, Equations, Number and Variables, chunks held by participants in common before the institute the chunks Relationships, Coordinates, Solving, Formulas and Types of graphs were also on 5 or more participant maps after the institute. Of the six chunks that participants held in common before the institute, (namely Graphing, Functions, Patterns, Equations, Number, and Variables), only Graphing, Function and Pattern were found to be significantly included on more participant maps after the institute. Recall that the main content focus of the institute was pattern, function and graphing and the relationships among these topics. Table 5 illustrates that at the beginning of the institute that 24% of the participants used pattern , 28% used function, and 34% of participants used graphing on their maps. By the end of the institute 80% used patterns, 84% used functions and 100% used graphing as chunks on their maps.

Insert Table 5 here

Elaboration of Participant Content Learning

We performed a content analysis (Weber,1990) of participants' responses to the prompt:

Choose a concept that appears on both maps. Write a paragraph or two explaining what more you understand more about the concept than before you began the institute, and the prompt *If possible, choose a concept that is on your second map but not on your first. Write a paragraph or two explaining what you understand about that concept.*

The purposes of this part of the analyses were twofold: (1) to verify that the content chunks portrayed on participant's maps was the actual content that participants themselves thought they had learned; and (2) to explore the participants' self-reported reasons for growth in content understanding.

Functions, patterns, formulas (and the connections between them) and graphing were concepts that all participants reported in response to either one of these prompts. Of the 25 participants, eight claimed that they had a better understanding of the function concept through their working with *patterns* during the institute.

> *A concept on my second map was T-charts. I really enjoyed trying to figure out the relationship between the x and the y by looking at the patterns.*

Two participants mentioned that, as visual learners, working with the graphing activities had given them deeper insight into functions and patterns.

> *Graphing was not on my first concept map. For some reason I did not make the connection between algebra and graphing. I now understand that graphing can serve as another tool to finding a pattern, predicting a pattern, experiencing a pattern and not just showing results of random information.*

Four of the participants stated that they had put function or formula on their pre map but had little insight in to the meaning of the concept before the institute.

> *In school, as I took math classes, I was shown what I had to do with certain numbers in a formula. I understand formulas better now because in this institute we used patterns and T-graphs to watch the formula come alive.*

A further four participants mentioned that the work done with patterns during the institute had helped them better understand differences between the graphs of certain families of functions such as linear, quadratic and exponentials.

> *I have a better understanding of function since I've revisited the concept for the past 8 days. I have also had practice with graphing functions and comparing differences between linear and exponential growth. I have some great lessons to take back to my classroom.*

Many participants mentioned more than one concept or said that understanding of one had led to further understanding of another.

> *Graphing shows up on both maps. I increased my knowledge of graphing by looking for patterns in In-Out tables and identifying patterns as a function of a number.*

> *Exponents. I first used the term incorrectly. The graphing activities increased my understanding. I was able to relate the concept of exponential growth to information I have been learning about finances, residual income, etc.*

The seven participants who did not mention functions or patterns in response to the first prompt, all chose it as the concept that was on their post map but missing from their pre map. In general, these teachers made connections between patterns or equations and their representations as functions, thus better understanding the function concept. Teachers noted growth in understanding of linear, quadratic, and exponential functions and why the respective graphs form their particular shapes.

> *The work we did each day on patterns, functions and algebraic thinking helped me understanding how to develop functions and use real situations to see how they are used in our daily life.*

The participants learned to interpret the graphs and connect the forms the graphs were taking to the function or equation by noting the T-chart representation of the functions.

Further, teachers realized that algebra and geometry are related topics and wrote that the activities during the workshop helped them to see the relationship between the two.

> *I truly thought until now that geometry and algebra were separate. As a visual and kinesthetic learner I'm so excited with this new discovery. I've always been strong in algebra and challenged in geometry so I'm learning to strengthen my geometry by understanding algebra.*

Conclusion

The results reported above indicate that participants' understandings of algebra changed as a consequence of participating in the project's Summer Institute. The content analysis in addition to the structural analysis of participant maps indicated to us how the participants' subject matter knowledge had increased in terms of breadth, depth and connectivity. These techniques allowed us to examine how adaptation of pre-institute knowledge occurred in order for new knowledge to be assimilated into the network structure. Further, for both of these analyses, participants' own reflective writings were used to triangulate results. Participants themselves felt that they had gained a greater understanding of concepts, in particular, graphing, functions and patterns. They further felt that their maps represented the greater connectivity of their understanding of algebra concepts. Thus we feel confident in recommending concept maps in conjunction with reflective writing as a means of assessing participants' understanding in specific mathematics topics, especially those that may result from professional development.

In general we found the use of concept maps as a way of assessing change in teachers' knowledge networks did not violate the principles under which the project operates. In this context, we found that the use of concept maps as a way of assessing change in teachers' knowledge networks did not violate the principles under which the project operates. Concept maps are a respectful approach to assessing teacher's growth in mathematical understanding because they provide:

1. Opportunities to build participants' collective understanding and vocabulary of algebra at the beginning of an institute and hence enhance the building of community.
2. A valuable learning activity in which participants reflect on their own understanding .
3. Validation of the mathematical knowledge that each participant holds.
4. An assessment of important mathematics consistent with the learning goals of the institute.
5. A means of documenting growth among a group of participants with radically different understandings of subject matter.

In this study we used triangulation of concept map data with the reflective writings of participants to establish their use as a credible method to measure change in important mathematical content as described in this paper. To give more credence to the use of concept maps as a scientific measure of growth in mathematics content understanding in general, further study that tests their concurrent validity is needed. For example, structural variables could be correlated with task-based assessments that require participants to connect learned concepts to other areas of mathematics.

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Figure Legends

1. Five Principles Underlying Choice of Assessment for Teachers' Mathematical Understanding.
2. Terms associated with Structural Concept Map Variables.
3. Six Structural Variables.
4. Pre-Institute Concept Map depicting algebra understanding from Participant 7.
5. Participant 7's Post Map.
6. An Example of a Participant's Pre-concept Map Illustrating an Extensive and Connected Group of Algebraic Concepts.

Figures

Principles of Equitable Assessment

1. Use of assessment in professional development should not interfere with the commitment to build a trusting and caring community of educators.

2. The assessment process should be, in itself, a valuable learning experience.

3. The assessment process should be respectful of the teacher and not invalidate the mathematical knowledge they hold.

4. The assessment should 'measure what counts' in terms of mathematical content knowledge.

5. The assessment instrument should serve to contradict the tendency of mathematics assessment to preserve the status-quo. (D'Ambrosio (1985) writes ". . . mathematics has been used as a barrier to social access, reinforcing the power structure which prevails in the societies [of the Third World]. No other subject in school serves so well this purpose of reinforcement of power structure as does mathematics. And the main tool for this negative aspect of mathematics education is evaluation."

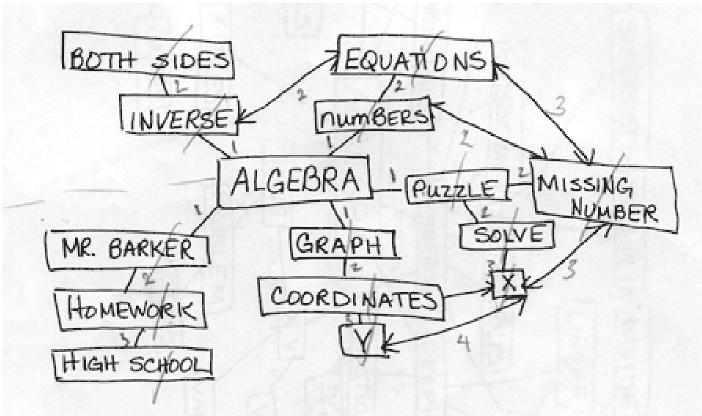
Figure 1. Five Principles Underlying Choice of Assessment for Teachers' Mathematical Understanding.

TERM	MEANING
Root	The main/first concept on a map
Concept	One idea depicted on a map by a circle or box
Link	A connecting line between two concepts
Successor of a concept	Any concept that is joined to a previous concept by a link
Depth of Concept Map	The length of the longest chain on the map
Level	A number, *X* , representing the concepts on the map that are *X* links away from the root
Width	The number of concepts on the largest level
Chunk	A group of linked concepts for which the leading concept has at least two successors
Crosslink	A link that connects two separate chunks together to indicate a relationship between them

Figure 2. Terms associated with Structural Concept Map Variables.

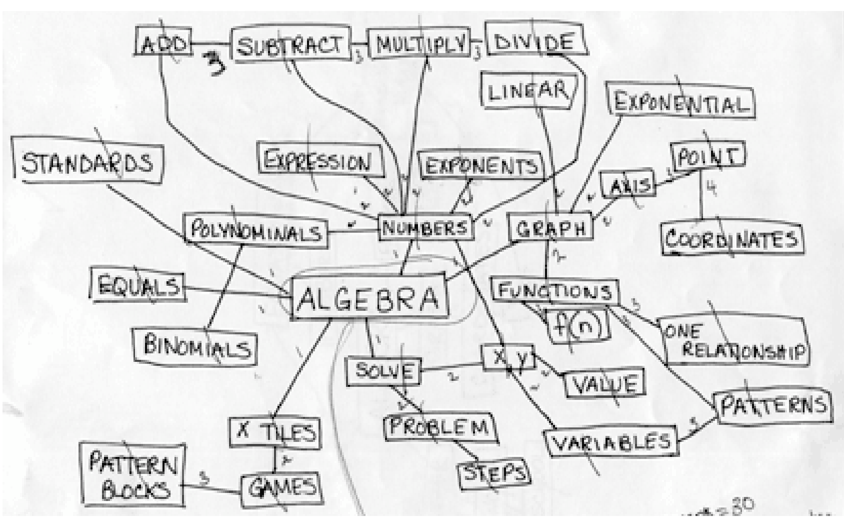
TERM	DEFINITION	COMMENT
Concept Number	Total number of concepts on each map	The number of concepts on a map is assessing the amount of algebra terms that a person knows.
Width	Greatest number of concepts at one level on the map; the widest point on the map	The width captures the breadth of knowledge.
Depth	Length of the longest chain on the map	The depth reflects the depth of a person's knowledge.
HSS	Width + Depth	HSS assesses the complexity of the map structure.
Chunk Number	Total number of chunks on each map, where a chunk is defined by any node that is linked by two or more concepts	Assesses the extent to which concepts and thoughts are interconnected, demonstrating connectivity of the structure of mathematical understanding.
Crosslink Number	The total number of crosslinks on each map where a crosslink is defined as a link between two chunks.	

Figure 3. Six structural variables.



Concept Number: 14
Width: 6
Depth: 3
HSS: 9
Chunks: 1
Crosslinks: 6

Figure 4. Pre-Institute Concept Map depicting algebra understanding from Participant 7.



Concept Number: 30
Width: 17
Depth: 4
HSS: 21
Chunks: 5
Crosslinks: 5

Figure 5. Participant 7's Post Map of algebra understanding.

Figure 6. An Example of a Participant's Pre-concept Map Illustrating an Extensive and Connected Group of Algebraic Concepts.

Table 1. *Comparison of Pre and Post Structural Variables and Chunk Names for Participant 7*

	Concepts#	HSS	Chunk#	Chunk names	Link#
Pre	14	9	1	puzzle	0
Post	30	21	5	Numbers, graph, functions, (x,y solve	2

Table 2. *Descriptive Statistics for the Structural Variables in Teachers' Concept Maps*

	Mean		Standard Dev.		Range	
N=25	Pre	Post	Pre	Post	Pre	Post
Concept Number	24.2	39.6	10.9	14.2	5-53	21-95
HSS	15.9	21.1	5.5	6.8	2-29	6-41
Chunk Number	6.3	13.2	4.6	9.8	0-18	5-53
Crosslink Number	1.9	3.3	2.0	2.9	0-8	0-11

Table 3. *Ranges and Difference Scores for the Structural Variables*

	Mean Differenc Score	Range of Difference scor	Number of Participants whc score increased	Number of Participants whc score decreased
Concept Number	15.34	-5 - 42	23	2
HSS	5.20	-5 - 14	22	3
ChunkNumber	7.14	-2 - 35	23	2
Crosslink Number	1.50	-3 - 10	22	3

*included in column 3 for Crosslink# are the participants whose number of crosslinks remained the same.

Table 4. *F Tests for Pre/Post Change of Structural Variables*

Variable	df	F	P value	η^2
Concept Number	1	21.46	<.01	.29
HSS	1	11.87	<.01	.18
Chunk Number	1	12.70	<.01	.19
Crosslink Number	1	5.13	<.05	.09

Table 5. *Comparison of Chunks on Pre and Post Maps*

Name of Chunk	Number of participants that listed content chunk on their map (% of participants)		Pearson Chi Square	P value
	Pre	Post		
Graphing	**9 (36%)**	**25 (100%)**	**23.5**	**.00**
Functions	**7 (28%)**	**21 (84%)**	**15.9**	**.00**
Patterns	**6 (24%)**	**20 (80%)**	**18.1**	**.00**
Equations	10 (40%)	9 (36%)	.34	.80
Number	9 (36%)	9 (36%)	0	1
Variables	6 (24%)	7 (28%)	0	1
Relationships		**6 (24%)**	**6.8**	**.02**
Coordinates		**6 (24%)**	**6.8**	**.02**
Solving		**6 (24%)**	**6.8**	**.02**
Formulas		**6 (24%)**	**6.8**	**.02**
Types of Graphs		**5 (20%)**	**5.5**	**.05**
Order of Operations	4 (16%)		4.4	.1

* Bold items are statistically significant.