Beginning Teachers' Knowledge Networks in Statistics: How They Develop During an Intensive Summer Institute

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ABSTRACT

The study was set in the context of a 5-year National Science Foundation sponsored teacher retention and renewal professional development program in which both beginning and experienced teachers are supported in the profession as they increase their understanding about mathematics, their ability to implement effective mathematics practices in their classrooms and their knowledge of working with English Learners. Data were collected from twenty-six teachers in their first five years of teaching as they participated in a 5 day summer institute. The mathematics theme for the week was statistics. Concept map data and responses to reflective prompts designed to elicit participant's understandings of statistics concepts were collected before and after the institute. Results showed that teachers' self-reported change indicated more complex and connected knowledge structures after the institute. These findings were reflected by the concept map measures. In addition pattern-matching techniques were used to show that participants were able to adapt their knowledge networks to incorporate important aspects of statistics into them. Concept maps are recommended to other leaders of mathematics professional development as a means of assessing areas of change in content knowledge, especially when the participants represent a wide array of mathematics backgrounds.

INTRODUCTION

Traditionally, there has been little relationship between the mathematics taught in school classrooms and mathematics “per se” as done by a practicing mathematician. In a traditional classroom students learn rules and formulas presented by the teacher, are expected to memorize these rules, spend many hours practicing using them and rarely apply them to real life phenomena. Students in these classrooms often come away with the impression that doing mathematics is a rote and mechanical activity that is prescribed, all authority for mathematics lies in a text book or as privileged knowledge inherent in the classroom teacher, and that memorization of formulas and procedures is the way to succeed mathematically (Hough, 2001). As a result, many students are turned off early from mathematics, they often see it as a discipline only understandable by a select few and they take as little as they can when given the choice (Skemp, 1987). In contrast, most mathematicians and others who use mathematical ideas see it as a creative, sense making activity that entails interpretation, hypothesizing, effort and guess work.

During the 80's and 90's national proposals for educational reform have called for quite a radical transformation of the way that school mathematics is taught (National Council of Teachers of Mathematics (NCTM, 2000). In such a classroom the teacher's role changes from being the sole authority of knowledge to a facilitator whose role is to help students construct concepts and understanding. The focus on students practicing skills and procedures is exchanged with students solving non-routine, open-ended problems and developing their own solution strategies. And individual seatwork is substituted with cooperative group work and discussion of mathematical ideas.
These calls to reform have been motivated by both research about how a child learns and by a view of mathematics as described by Hersh (1986).

> *Anyone who has ever been in the least interested in mathematics, or has ever observed other people who are interested in it, is aware that mathematical work is work with ideas. Symbols are used as aids to thinking just as musical scores are used as aids to music. The music comes first, the score comes later. Moreover, the score can never be a full embodiment of the musical thoughts of the composer. Just so, we know that a set of axioms and definitions is an attempt to describe the main properties of a mathematical idea. But there may always remain an aspect of the idea which we use implicitly, which we have not formalized because we have not yet seen the counterexample that would make us aware of the possibility of doubting it.* (p.19)

Thus, in a reform classroom the focus of the mathematics curriculum has been shifted away from isolated facts and procedures and toward mathematics as a process of understanding interconnected concepts and ideas by using reasoning, problem solving and communication. The "doing" of mathematics as advocated by these reforms consists of students working both alone and in groups solving authentic mathematics problems, in which the inherent mathematics subject matter is connected to the student's own lives and to their prior knowledge. Ideas and hypothesis are justified and challenged and student misconceptions are analyzed and used to further collective understandings within whole class discussion. The National Council of Teachers of Mathematics recognizes the considerable importance of the classroom teacher in implementing these reform principles in the classroom (NCTM, 2000). Considering these attempts to change the culture of mathematics classrooms and the recent advent of rigorous content standards being implemented throughout U.S. education (for instance, CDE, 99) there has been much discussion on what kinds of mathematical knowledge teachers need in order to teach mathematics in line with NCTM vision (Ball 2000, Hill 2004).

Context for Study

The study was set in the context of a five-year National Science Foundation sponsored teacher retention and renewal professional development program in which both beginning and experienced teachers are supported in the profession as they increase their understanding about mathematics, their ability to implement effective mathematics practices that are based on these recommendations of the National Council for Teachers of Mathematics (NCTM, 2000) in their classrooms and their ability to teach mathematics to English Learners (ELs). A leadership team of six university mathematics educators and school-based lead teachers works with groups of experienced teachers in summer retreats and series of academic seminars, who in turn design professional development opportunities in mathematics, teaching and learning, using a mentoring model, for the beginning teachers in their schools and districts. During the past 4 years the project has impacted over 420 participants from 80 schools within ten school districts in Southern California.

In this study we focused on the understandings of mathematics of a small group of the beginning teachers in the project as they participated in an intensive summer workshop in statistics that was offered by a leadership team of university based and school based mathematics educators. During their five days together at the institute, the twenty-six teachers participated in daily mathematics activities designed to directly affect their understandings of statistics subject matter and its importance in the K-12 curriculum. The statistical concepts focused on were basic and focused on measures of central tendency, methods of graphing and displaying data, probability, sampling and the misuse of statistics, but were designed to allow participants to develop deep understandings of them. Activities that fostered connections between statistics and other strands of mathematics, such as number sense and algebra, were included. Special attention was given to allowing participation in activities regardless of prior knowledge of concepts. In addition teachers participated in four grade level specific discussion groups that were designed to encourage them to reflect on their learning for that day and to apply it directly to their grade level. These discussions enabled them to make connections between the mathematics that they teach and the concepts that they were learning (or re-learning).

In their recent review of the literature on “Mentoring for Standards Based Reform”, Wang and Odell (2002) outline facets of “knowledge, skills and dispositions” (p 485) that beginning teachers should be expected to learn in mentoring programs that focus on the implementation of such reform. Three of these are the focus of the broader evaluation study that is being conducted for our project:

1. A relevant disposition towards *standards*-based reform that includes a teacher’s beliefs about teaching, beliefs about learning and beliefs about mathematical knowledge.
2. The development of deep and flexible conceptual understandings about subject matter.
3. Pedagogical learner knowledge, relevant to diverse learners, to teach in a way consistent with the standards.

In this paper the focus is on 2: Beginning teachers development of deep and flexible conceptual understandings about subject matter.

How Are Deep and Flexible Conceptual Understandings Developed?

Jean Piaget, a genetic epistemologist, who is considered one of the first *constructivists* (Von Glasersfeld, 1997) purports a theory of the development of *knowing* that is in line with the reformers view of how mathematics should be taught. Constructivism is a theory of learning that views all knowledge development as an adaptive function in which an attempt by a leaner to adapt to his/her environment is made through the processes of assimilation and accommodation of the learner's current cognitive structures. These cognitive structures are created from schemas, primitive mental organizations that the learner created in order to understand the world or a certain portion of it. As schemata become more complex they form cognitive structures. As the structures become more complex they become organized in a hierarchical manner from general to more detailed. As new understandings come to the learner's attention they are either assimilated into a pre-existing schemata, or by accommodation, when the new understanding of information does not fit into any existing schemata and puts the knower into a state of disequilibrium. According to Piaget, this is when real learning occurs—when existing schematas need to be re-arranged to accommodate the new information. Thus a learner does not necessarily grasp a new set of ideas all at once but attaches new concepts to part of already existing cognitive structures (assimilation) and sometimes, as in adaptation, completely re-arranging the structures in the process.

During this intensive summer institute participants have many opportunities to learn statistics concepts in this manner and build more robust cognitive structures about statistics by "doing" authentic mathematics activities and by having time to reflect and discuss what they have learned and how such learned concepts can be applied in their classroom. Typically RENEW participants arrive at professional development initiatives with diverse backgrounds in content knowledge. Some are high school teachers with extensive content knowledge, others are elementary teachers with limited understandings of mathematics. These teachers also vary in their confidence in regards to doing mathematics. Knowledge that participants bring with them is honored and project staff design activities to allow for participation by teachers with varying degrees of prior knowledge. The purpose of the activities is to facilitate growth from where each teacher started. As a consequence, in this study we use pre and post concept maps along with reflective writings to attempt to capture a teacher's cognitive structure of statistics concepts and how his/her structure might change as a result of participation in authentic mathematical activities.

Why concept maps?

According to Novak, (1984) a concept map is a two dimensional drawing which purports to represent a participant's cognitive network related to a specific topic. In particular the map represents concepts that the participant connects to a central concept (such as *Algebra* or *Statistics*) and the relationships among those topics. Individual concepts are represented by circles on the map and are connected to the main concept and/or other related concepts by lines or arrows. Concepts close to the main idea or topic and drawn closer to it and those further away represent ideas connected less strongly to the main one. Content chunks on a map form when a particular concept has two or more direct descendents and links between content chunks are represented by multi-directional arrows. We used concept maps as a data collection technique in this study because we believe that they reflect what a participants prior concept knowledge better than other methods; (b) they show how a person connects related topics to each other and makes connecting links between sub-topics related to the main concept and (c) there are both structural and content-related methods of analyzing maps that show possible change in a participants cognitive network. Figures 1 and 2 below are of participants A's pre and post institute concept map which are shown to illustrate how learning took place for her. For example, *Graphing* was a concept on the pre-map in figure 1. In figure 2 new knowledge of graphing techniques have been assimilated into participant A's cognitive network to form a content chunk. Similarly the "mathematics" concept has become a content chunk in the post map as new concepts and understandings are acquired. Additionally, the *Research* content chunk on the pre-map, has been re-arranged and combined with the concepts of surveys and questions to accommodate some new understandings and the creation of a new "research" chunk on the post map.

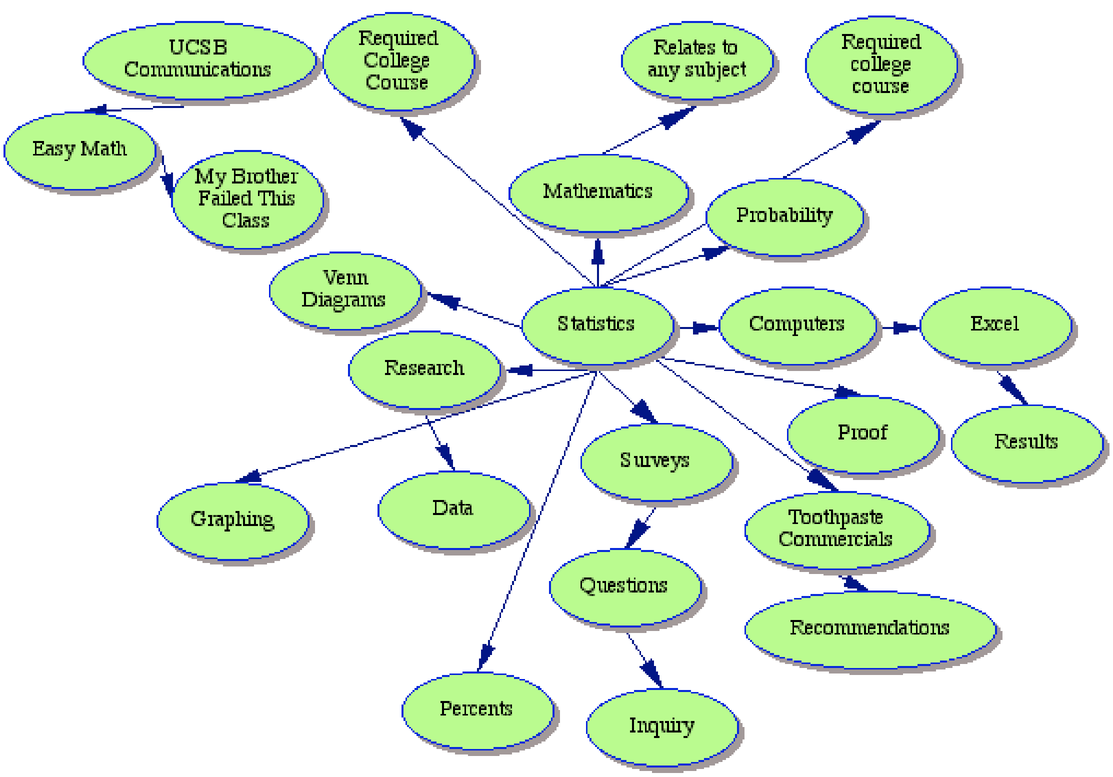

Figure 1. Participant A's Pre Statistics Map

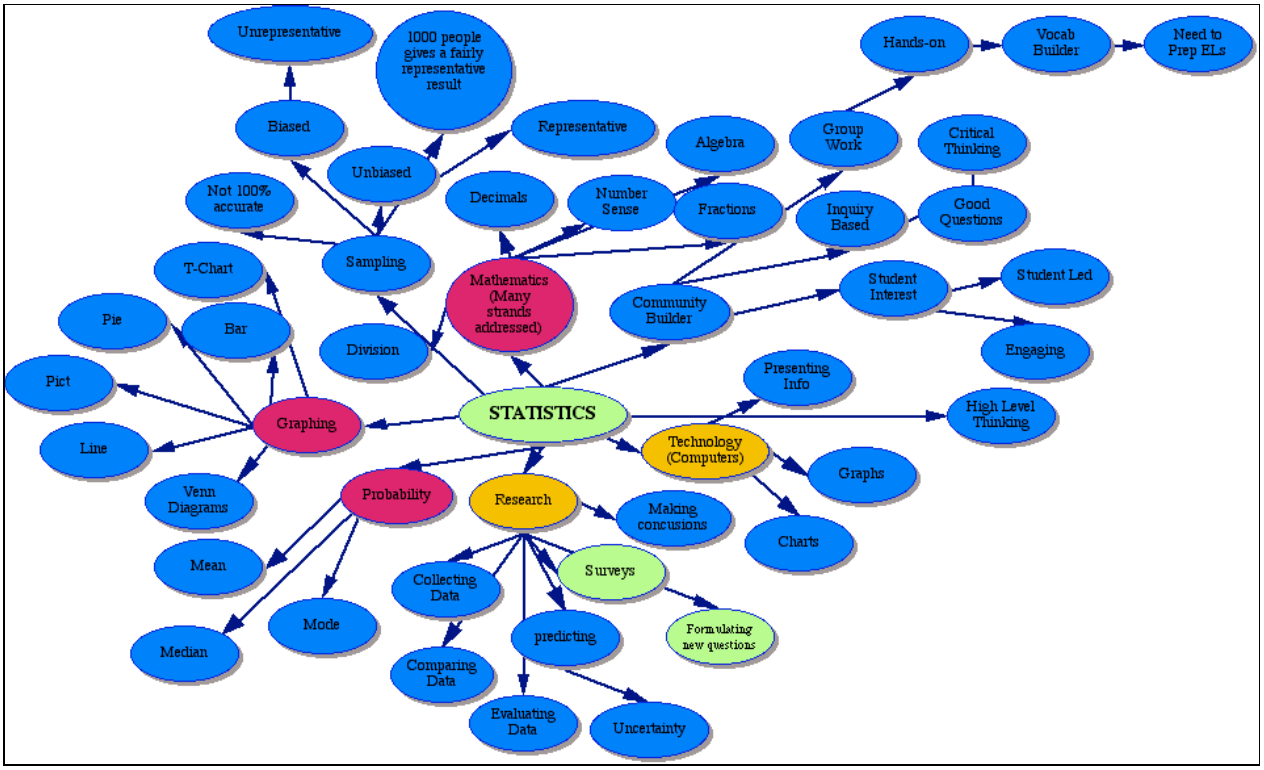

Figure 2. Participant A's Post Statistics Map

In general, on the post map, the blue concepts are new concepts that have been added, the pink show chunks were assimilation have taken place and the gold show accommodation.

METHODOLOGY

The twenty-two beginning teachers who participated in the five-day summer institute on statistics and its teaching agreed to participate in this study.

Data Collection

Pre and post institute data were collected using concept maps and reflective writings. On the first day of the institute participants were given a short introduction to concept maps and their multiple uses in education by a project leader. The group then collectively generated a concept map about teenagers in order for them to gain comfort in drawing one. A concept map on fractions was then shown as another example. Participants were then given 15 minutes to draw their own maps on statistics. After fifteen minutes participants paired up and had a dyad[1] in response to the prompt: *How did you feel about doing this activity?* Next a whole class discussion ensued in which teachers generated a common list of vocabulary/concepts that they associated with statistics (see figure 3 below).

> *Mean, median and mode (shared by one participant as "the big three"), data, standard deviation, graphing and displaying data, probability, error, predicting, sorting, classifying, research, significance, bad professors, sports, gambling*

Figure 3. Common list of Vocabulary Concepts for Statistics.

On the last day of the institute, teachers were again asked to draw a concept map about statistics. They were instructed to work alone and to initially not consult their pre-map. They were given 15 minutes to complete this task. They were then asked to look at their map, compare it to their pre map and to respond in writing to the following prompts:

1. What do you think the map showed about your learning of statistics?
2. What do you think that concept maps don't show about your learning?
3. Pick a concept that is on both your pre and post map and write a paragraph about how your understanding of that concept has changed.
4. Pick a concept that is on your post map but not on your pre-map and write a paragraph about what you have learned about that concept.

ANALYSIS

Data were analyzed using both structural/numeric (Novak & Gowind, 1984) and content analytic (Morine-Dershiker, 1993) techniques. First variables representing the number of concepts, the breath and depth, the number of chunks and the number of links on each map were created. To illustrate these variables consider figures 1 and 2 above. The number of concepts on this map (CONCEPTS) is 22. A chunk, is any concept that has at least two direct descendants. For instance in figure 1 there are no chunks, but in figure 2 there are Eight: Graphing, Probability, Research, Technology, Community Building, Student Interest, Mathematics, and Sampling. The breath of a map is how many concepts there are on the largest level. Participant A's pre map has four levels with 12 concepts on level 1. This is the breath of the map. The depth is the number of levels, so in this case it is three. The Hierarchical Structure Score (HSS) is obtained by

[1] A dyad is a two person activity in which one person speaks for a set amount of time while the other person listens without interrupting. The pair then switch and the listener gets a set amount of uninterrupted time to talk (see Weissglass, 1990)

adding together the breath and depth of a map. For participant A, HSS is 16. A link is when there is a multi-directional arrow drawn connecting any two chunks on a map.

A Multiple Analysis of Variance was then run using the statistical program SPSS, with these four variables as dependent variables to test whether there was any pre/post change in the maps.

Next, content approaches were used to test for the inclusion in maps of important content that was the focus of this Statistics Institute. The number of participants that included each content focus on their maps as Concepts and as Content Chunks was tallied. For instance, in figure 1 participant A included graphing, data and probability on her pre maps as concepts. On her post map she also had graphing as a Content Chunk. Chi Square Statistics were then calculated to test for pre and post differences for each of the content foci.

Finally we used a sub-sample of purposefully selected participant maps along with their reflective writings to both triangulate the results of the above analyses and to explore further, how participant maps changed as a result of this five-day institute.

RESULTS

Structural Analysis Results

Table 1 below illustrates large gains from pre to post maps on all four of the variables. The number of concepts on the average participant's map increased by 11 concepts, almost a full standard deviation; the Hierarchical Structure Score increased by 6 concepts –about three quarters of a standard deviation; the number of chunks increased by 2, about half a standard deviation and the average number of links went from 0 to 1, a whole standard deviation. A MANOVA indicated that overall these pre-post differences were statistically significant, with an effect size considered large in social science research ($F(5, 36)=2.9$, $p = .02$, $\square^2=.28$).

Table 1. *Descriptive Statistics for Variables*

	PRE		POST	
	Mean	Std. Dev.	Mean	Std. Dev.
CONCEPTS	24	8.8	35	12.2
HSS	15	4.8	21	8.5
CHUNKS	4	4.6	6	3.7
LINKS	.1	.4	.9	1.0

Table 2 displays the results of the subsequent univariate tests, ran to determine which of the four variables was contributing to this effect. Results showed that all four dependent variables contributed to this statistically significant difference with medium to large effect sizes ($\square^2$)

Table 2. *Tests of Between-Subjects Effects*

Dependent Variable	Type III Sum of Squares	Mean Square	F	Sig.	$\square^2$	Observed Power
CONCEPTS	1152.4	1152.4	10.2	.00	.20	.88
HSS	421.2	421.2	8.8	.00	.18	.83
CHUNKS	48.2	48.2	3.8	.05	.08	.47
LINKS	6.1	6.1	5.1	.03	.11	.59

Few sentences go here to connect results to next sentence
In summary then, we claim that as a result of participating in the Summer Institute in Statistics Teaching and Learning, participants knowledge structures of statistics not only grew in terms of the amount of concepts they knew but became more complex and connected.

Content Analysis Results

Table 3 below illustrates which new concepts and connections participants made on their post maps. The bolded items are those that attained statistical significance. A few interesting results need to be picked and a sentence written about them. For instance notice that all of the participants included graphing as an important statistics concept on their post maps.

Table 3. *Frequencies of Selected Concepts and Chunks Appearing on Participant Maps*

	Appeared on Map as Concept				Appeared on Map as Chunk		
	% Participants				% of Participants		
	Pre	Post	P value of pre-post change		Pre	Post	P value of pre-post change
Measures of Central Tendency	52%	71%	.20		**5%**	**33%**	**.02**
Graphing	**62%**	**100%**	**.00**		**24%**	**62%**	**.01**
Sampling	**0%**	**48%**	**.00**		**0%**	**29%**	**.00**
Probability	**47%**	**76%**	**.05**		24%	19%	.70
Bias	**10%**	**67%**	**.00**		10%	14%	.63
Data Collection	43%	48%	.75		5%	19%	.15
Data Analysis and Interpretation	24%	48%	.10		**10%**	**33%**	**.05**

Few sentences go here to connect results to next sentence
In summary then we claim that during the institute participants gained a more connected and complex understanding of the important statistics topics taught.

How Participants Incorporate These New Understandings Into Their Existing Knowledge Networks.

We purposefully selected three participant pre and post maps and use these, along with these participant's reflective writings, to illustrate how new understandings were incorporated into the participant's prior cognitive knowledge network. Table 4 below gives descriptive statistics for these participant maps.

Table 4, Descriptive Statistics for Maps of Purposefully Selected Participants

	Concepts		HSS		Chunks		Links	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Participant A	22	50	16	37	0	8	0	0
Participant B	27	54	12	34	5	7	0	2
Participant C	41	33	28	28	8	6	3	3

Participant A (ne-ne)

These maps were displayed in figures 1 and 2 above and are displayed again below in figures 4 and 5 for completeness. Notice that she has more than doubled her number of concepts, chunks and her HSS score.

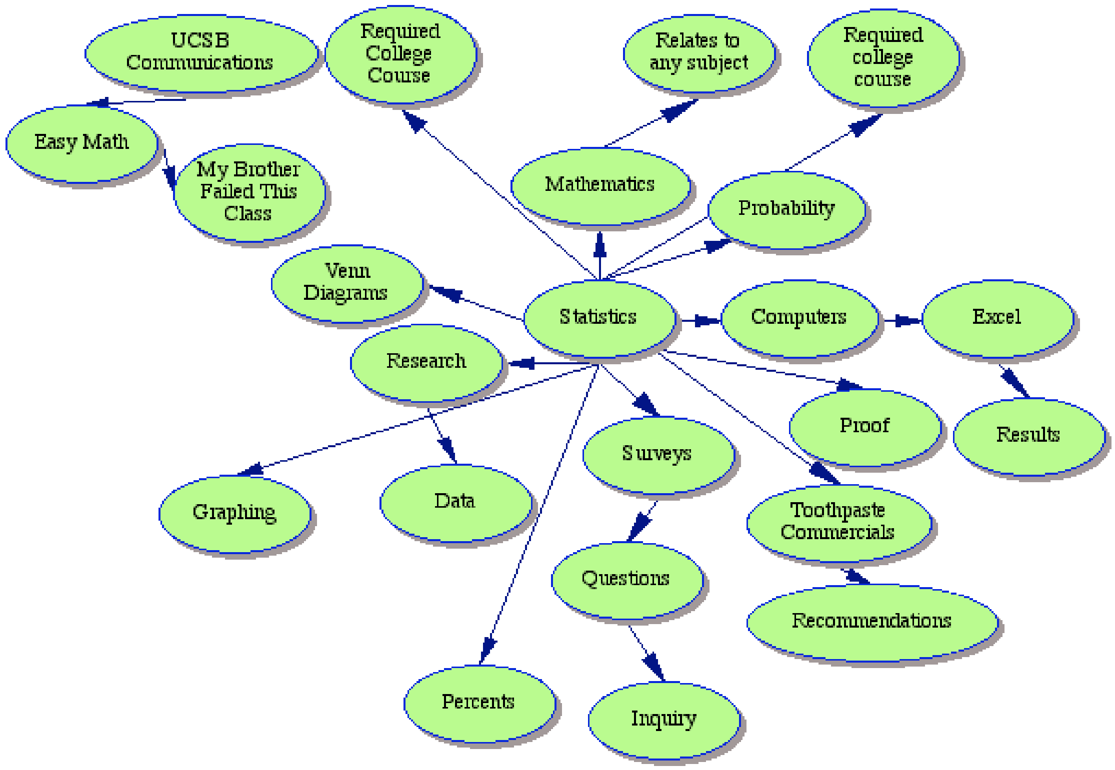

Figure 4. Participant A's Pre Map

The large increase in numbers on the variables from pre to post is representative of the majority of participants in this study. Participant A's map also shows how some new ideas have been assimilated into her cognitive network (see for instance the addition of different graphing techniques on her post map to the graphing node concept on the pre as well as the addition of the measures of central tendency to the probability node). Assimilation of ideas is represented on the post map in pink, with new concepts shown in blue. These two maps also illustrate instances in which a re-arrangement (accommodation) of ideas has taken place, for instance the research and technology nodes. These accommodations are illustrated in gold.

In her reflective writing this participant mentions her new understanding of graphing techniques:

> *After this institute I am now comfortable with how to represent my data in many different forms: pie, chart, bar graph, pictograph, line graph, Venn diagrams, etc. in relation to that, I feel I have come away with many ideas/topics that would be relevant to my grade level as well as have student interest.*

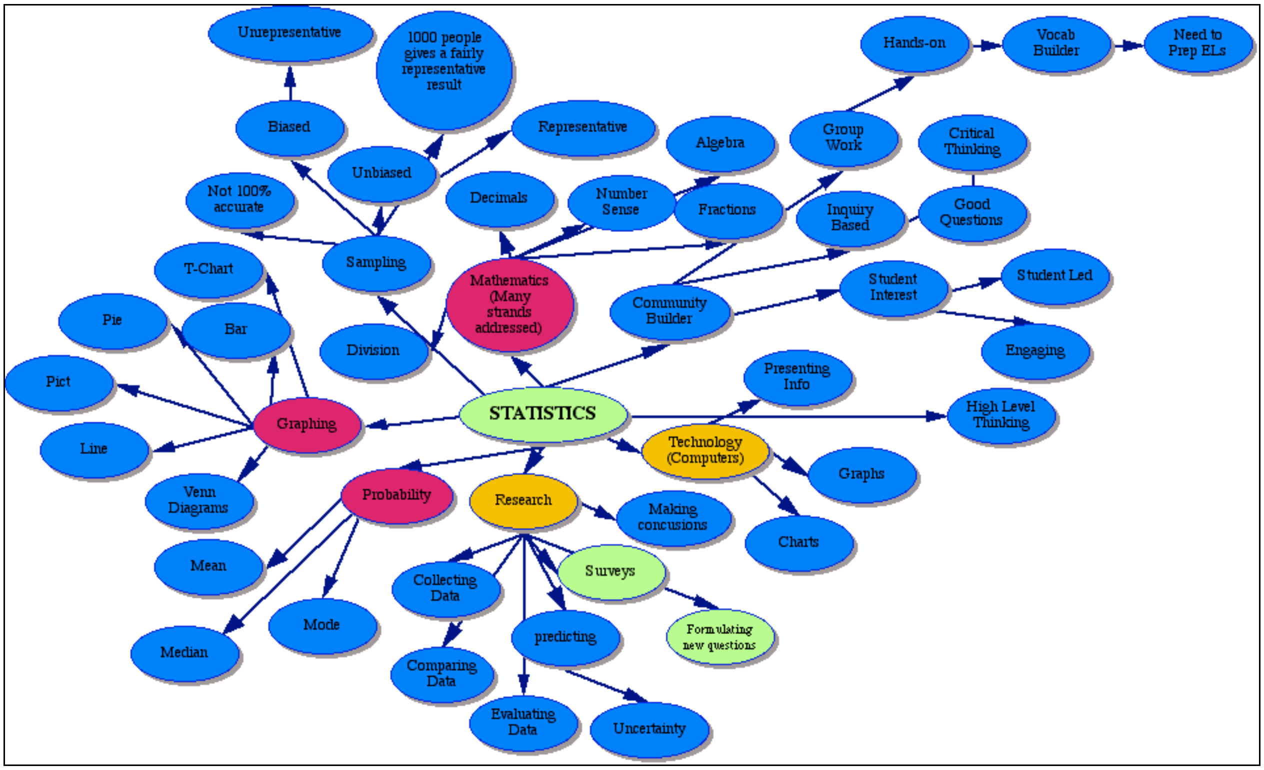

Figure 5. Participant A's Post Map

On the post map the *Community Building* chunk consists of concepts connected to student learning of statistics such as, student led, group work, critical thinking and good questions. This participant found that to be the most important aspect of her learning.

> *In regards to my new learned acquired understanding of statistics, the most profound ideas I have come away with is not only what the different realms of statistics are, but more importantly, how to integrate it into my classroom. I can honestly say statistics was not a topic I covered in my first year of teaching. I cannot dwell, on that fact, but rather improve in my second, third, fourth, and soon, years of teaching. I now can relate to why statistics is a great subject to start the year off with! Statistics is highly engaging, relative to student interest, can be student led, as well as being an inquiry type project.*

Participant B

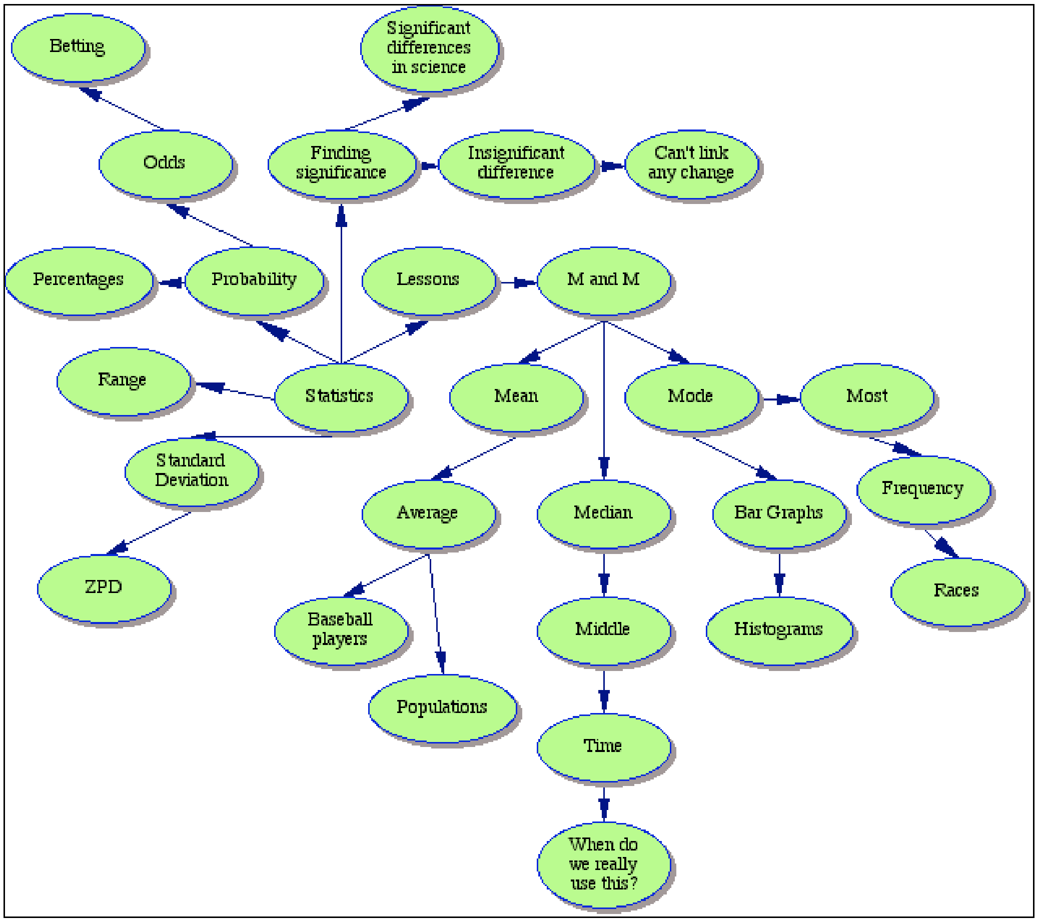

Figure 6. Participant B's Pre Map

My first graph does not have as many connections with the sub categories as the second one. The concept that I pick is measures of central tendency. This concept shows on both of my maps, what I understand more about then is that the mean could get affected by a very big number, while the median could stay the same. I also learned that as teachers we need to give the students the opportunity to find out the relationship between the measures of central tendency and how can they be used in real life.

Figure 7. Participant B's Post Map

Participant C

Participant C is an interesting case representing the few participants whose structural scores decreased on their post maps but who non-the-less made some interesting changes in their understandings. Notice the probability chunks on the pre and post maps. On the pre map the chunk consists of examples of adult instances that the learner associates with probability. On the post map the chunk consists of ways of teaching probability to children at her grade level. The concept chunk is colored gold since accommodation took place here. Further, on the post map the probability chunk is appropriately linked to the bias chunk. This is a chunk not present on the pre map.

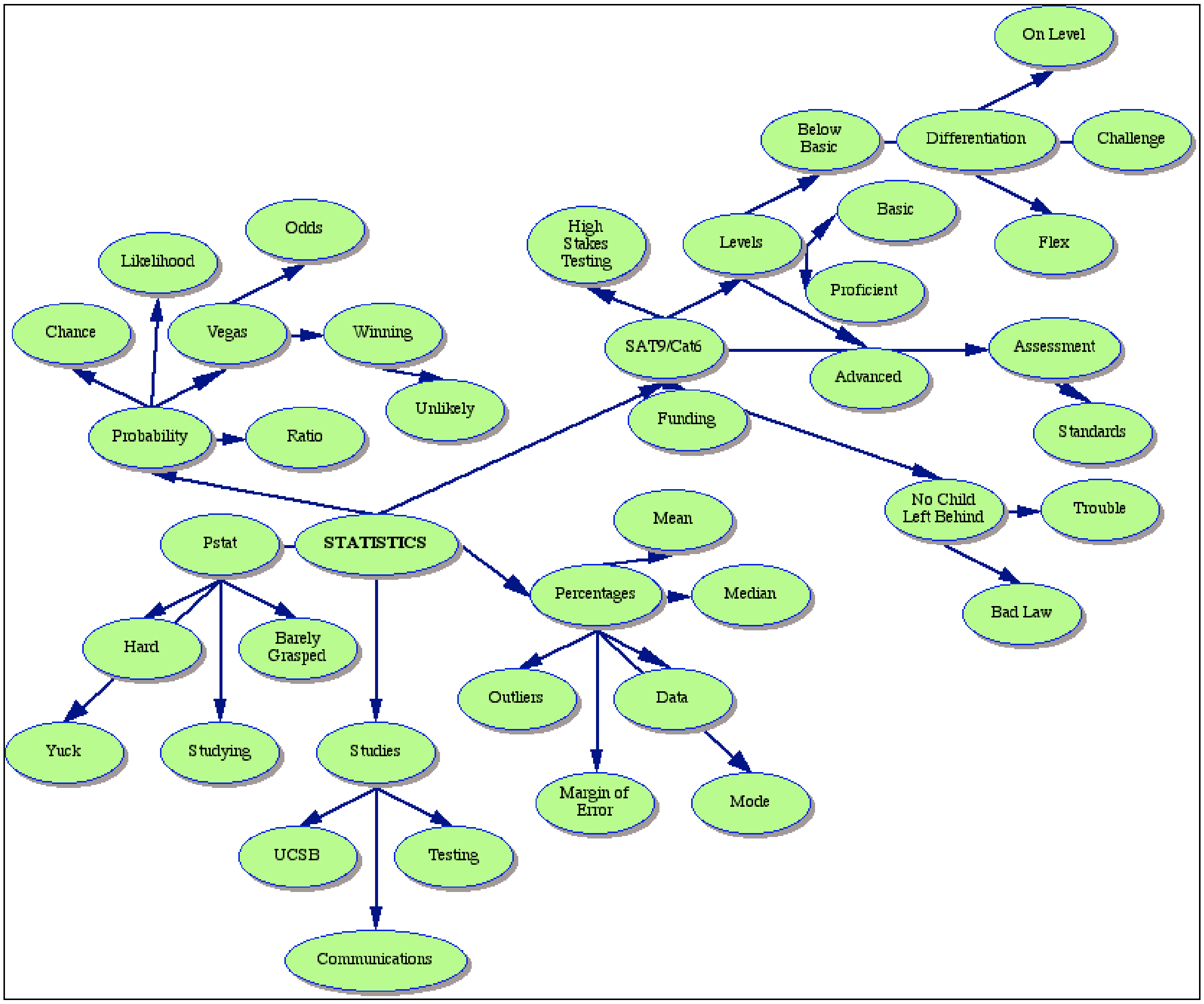

Figure 8. Participant C's Pre Map

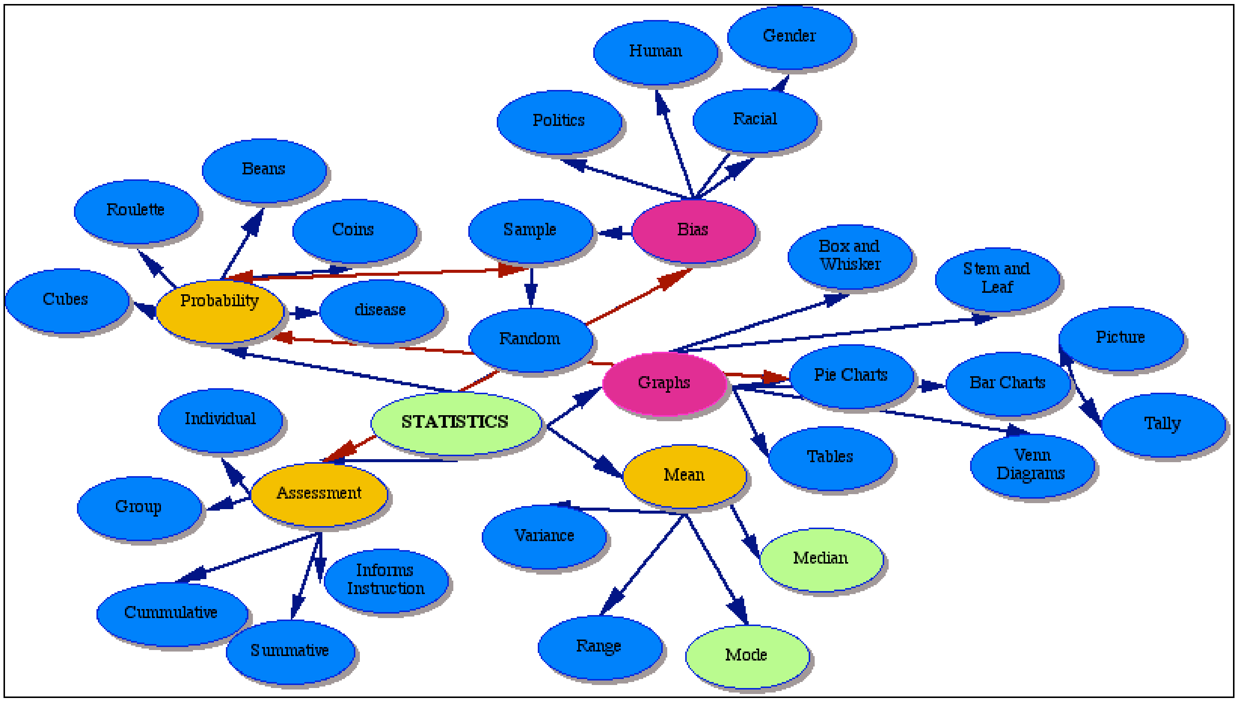

Figure 9. Participant C's Post Map

The participant herself chose "probability" and this link to focus on in her reflection:

> *On both of my maps the concept "probability" is present. I feel that through our activities this week I understand the way sampling can lead to conclusions in which you could predict the probable outcomes. Further more, these predictions can be tested to determine correctness. I feel I can take this idea to my students, and through appropriate questioning, because I understand the concept better, help them reason and construct their knowledge. On my second map, I have written about bias. This is an important idea when considering statistics that inundate us through the media. We explored bias by race, gender and human error. This has made me feel a new need to scrutinize statistics. In additions, I should in turn teach my students to be savvy also. In closing, I feel that I can really go back to the classroom and teach probability and statistics effectively. At the beginning of the week, I could not say that with any conviction.*

Sixty percent of the participants echoed the emphasis on children's statistics in their reflective writing, stating that through the classroom connections made that they now feel prepared better to teach appropriate statistics concepts to students at their respective grade levels. Below are two examples:

> *The most profound thing I have come away with is not only what the different realms of statistics are, but more importantly, how to integrate it into my classroom. I can honestly say statistics was not a topic I covered in my first year of teaching. I cannot dwell, on that fact, but rather improve in my second, third, fourth, and soon, years of teaching.*

What I would like to discuss is more about how I learned new and exciting ways to teach statistics rather then what I learned about the subject. I feel that I already had a good understanding on the subject. I've always enjoyed teaching stats, and now I will be even better at this.

On further examination, it seems that for Participant C, the drop in number of concepts represented a shift in focus from her experiences with college statistics (such as PSTAT, barely grasped, hard, yuck, studying, UCSB, testing) and experiences with standardized testing (such as SAT9/CAT6, No Child Left behind, differentiated instruction, bad law, trouble,…) to the mathematics and ideas learned at the institute.

Notice that the concept assessment on this participants pre map is associated with standardized testing and NCLB, whereas on the post map it is associated with concepts more in line with performance assessments.

Several other participants talked about their changed attitudes towards statistics subject matter as a result of becoming more comfortable with it during the institute.

Comparing both concept maps I noticed that my first map was filled with fear and my negative preconceived notions of statistics, my 2nd map is more enlightened because I was more focused on positive, meaningful activities that disseminated my fears, it was geared more towards group work, assessment, community and I have been able to see real life applications both inside and outside of the classroom.

One fourth of the participants focused attention on their pre maps on their previous negative experiences with Probability and Statistics (a required college course) rather than on their current experiences teaching statistics to their students. This changed as a result of the institute.

Prior to this week's activities, I attributed statistics more to a college level or advanced math level. However, now I understand and am excited to see how statistics applies to the elementary level of teaching and learning.

DISCUSSION

Offering professional development to teachers involves more than instructing them in subject matter using traditional lecture style workshops. If teachers are to teach in ways recommended by national standards based reforms–to give their students authentic mathematics activities that involve conjecturing, problem-solving, reasoning, and justification of ideas then they need to be given ample opportunities to experience mathematics subject matter themselves in this way. Further, traditional assessments of their learning and development are not appropriate. We have shown here that in the content of the development of subject matter, concept maps are an appropriate measure to gauge how participants of professional development activities are acquiring subject matter knowledge and further how they are visualizing that new knowledge in the context of the knowledge that they bring with them.

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